

# Effect of Variable Viscosity on Free Convection Flow in a Horizontal Porous Channel with a Partly Heated or Cooled Wall

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*A theoretical study of the effect of variable viscosity on the steady free convection flow in a horizontal infinite porous channel when a part of its bottom wall is heated is presented in this paper. The transformed equations are solved numerically using a finite-difference method. The effects of the Rayleigh number and viscosity parameter on the flow and heat transfer characteristics are discussed*

*Key words: porous medium, heated channel, variable viscosity, numerical method*

Convective heat transfer in saturated porous media has received considerable attention during the past several decades because of its wide range of applications in many chemical engineering and geophysical systems. These applications include underground spread of pollutants, packed bed reactors, porous insulation, beds of fossil fuels, nuclear waste disposal, porous catalysts, electrochemistry, food processing, energy efficient drying processes, nuclear and electronic equipment cooling, extraction of methane gas from clathrate hydrates, cooling of radioactive waste containers, crystal growth, to name just a few. The fundamental nature and the growing volume of work in this area is amply documented [1-7]. A rich variety of important analytical, numerical, and experimental results have been published on this topic and they are important to better understand the thermal convection inside porous cavities and channels. In this paper attention will be focused on the problem of steady free convection flow in an infinite horizontal porous channel with part of its bottom wall being heated when the dynamic viscosity of the fluid-saturated porous medium is variable [8]. It is to be noted that among the physical properties of a fluid which change with large variations of temperature is the viscosity [9]. For example water viscosity is  $\mu = 0.0131 \text{ g/cm} \cdot \text{s}$  at  $10^\circ\text{C}$  and  $\mu = 0.00548 \text{ g/cm} \cdot \text{s}$  at  $50^\circ\text{C}$ . Studies have shown that when this effect is included, the flow characteristics may be substantially changed compared to the constant viscosity case Basic equations [10, 11] etc.

Consider the two-dimensional steady-state free convection in a horizontal infinite porous channel with part of its bottom wall being heated or cooled as shown in figure 1, where both the upper and bottom walls are assumed to be impermeable. It is also assumed that the temperature

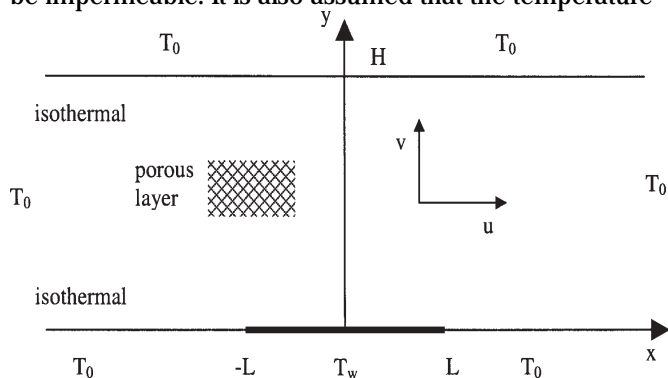


Fig. 1. Physical model and coordinate system

$T_w(x)$  on the central part of the bottom wall is higher (heated) or lower (cooled) than the constant temperature  $T_0$  applied on the wall away from the heated (or cooled) section. The temperatures of the upper wall and porous medium at large distances from the heated section are also assumed to be  $T_0$ . It is also assumed that the temperature  $T_w(x)$  of the heated section is symmetric, so that only the region  $x \geq 0$  needs to be considered. Mathematical model is described by continuity equation, Darcy law and energy equation. We introduce the following dimensionless variables

$$X = x/L, \quad Y = y/\varepsilon L, \quad U = u/U_0, \quad V = v/\varepsilon U_0, \\ \tilde{\mu} = \mu/\mu^*, \quad \theta = (T - T_0)/\Delta T, \quad \tilde{\mu} = \mu/\mu^*$$

where  $U_0 = (K/\mu^*)\varepsilon(\rho_0 g \beta \Delta T)$  is a suitable velocity scale,  $\varepsilon = H/L$  is the aspect ratio,  $\mu^* = \mu(T_0)$  is the characteristic dynamic viscosity, where  $u$  and  $v$  are the velocity components along the flow direction ( $x$ -direction) and normal to flow direction ( $y$ -direction),  $T$  is the fluid temperature,  $K$  is the permeability of the porous medium,  $\rho_0$  is the density when the temperature is  $T_0$ ,  $\beta$  is the thermal expansion coefficient. Further, using the dimensionless stream function  $\psi$  ( $U = \partial\psi/\partial Y$ ,  $V = -\partial\psi/\partial X$ ) the governing equations under the Boussinesq approximation are given by:

$$\varepsilon^2 \frac{\partial}{\partial X} \left( \tilde{\mu} \frac{\partial \psi}{\partial X} \right) + \frac{\partial}{\partial Y} \left( \tilde{\mu} \frac{\partial \psi}{\partial Y} \right) = - \frac{\partial \theta}{\partial X} \quad (1)$$

$$\varepsilon Ra \left( \frac{\partial \psi}{\partial Y} \frac{\partial \theta}{\partial X} - \frac{\partial \psi}{\partial X} \frac{\partial \theta}{\partial Y} \right) = \varepsilon^2 \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \quad (2)$$

Dimensionless parameter  $Ra = g K \rho_0 \beta (T_w - T_0) H^2 / (\alpha_m \mu^* L)$  is the Rayleigh number for the porous medium where  $\alpha_m$  is the effective thermal diffusivity of the porous medium. We notice that when the dynamic viscosity is constant ( $\tilde{\mu} = 1$ ), ((1) and (2)) reduce to those established in [8] for the steady-state free convection in a horizontal porous layer with a partly heated or cooled bottom wall.

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The boundary conditions of equations (1) and (2) are the following:

$$\psi = 0, \quad \theta = \begin{cases} \pm \theta_w(|X|) & |X| \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{on } Y = 0 \quad (3)$$

$$\psi = 0, \quad \theta = 0 \quad \text{on } Y = 1$$

$$\psi = 0, \quad \frac{\partial \theta}{\partial X} = 0 \quad \text{on } X = 0, \quad 0 \leq Y \leq 1 \quad (4)$$

$$\psi \rightarrow 0, \quad \theta \rightarrow 0 \quad \text{as } X \rightarrow \infty, \quad 0 \leq Y \leq 1 \quad (5)$$

The particular form of  $\theta_w(X)$  is that proposed [8] and that of  $\tilde{\mu}(\theta)$  is the one used [12]:  $\theta_w(X) = 1 - X^2$  for  $0 \leq X \leq 1$ ,  $\mu(\theta) = \mu^* \exp(b\theta)$  where the viscosity variation number,  $b$ , is positive/negative in case of a gas/liquid whose viscosity increases/decreases with an increase in temperature. One also notes that the Taylor series expansion for very small values of  $b$  leads to linear

$$2^\xi \ln 2 \left( \frac{\partial \tilde{\mu}}{\partial Y} \frac{\partial \psi}{\partial Y} + \tilde{\mu} \frac{\partial^2 \psi}{\partial Y^2} \right) + \frac{\varepsilon^2}{2^\xi \ln 2} \frac{\partial \tilde{\mu}}{\partial \xi} \frac{\partial \psi}{\partial \xi} + \varepsilon^2 \tilde{\mu} \left[ \frac{1}{2^\xi \ln 2} \frac{\partial^2 \psi}{\partial \xi^2} - \frac{1}{2^\xi} \frac{\partial \psi}{\partial \xi} \right] = -\frac{\partial \theta}{\partial \xi} \quad (6)$$

$$\varepsilon Ra \left( \frac{\partial \psi}{\partial Y} \frac{\partial \theta}{\partial \xi} - \frac{\partial \psi}{\partial \xi} \frac{\partial \theta}{\partial Y} \right) = \varepsilon^2 \left[ \frac{1}{2^\xi \ln 2} \frac{\partial^2 \theta}{\partial \xi^2} - \frac{1}{2^\xi} \frac{\partial \theta}{\partial \xi} \right] + 2^\xi \ln 2 \frac{\partial^2 \theta}{\partial Y^2} \quad (7)$$

$$\psi = 0, \quad \theta = \begin{cases} \pm 2^\xi (2 - 2^\xi), & 0 \leq \xi \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{on } Y = 0, \quad \psi = 0, \quad \theta = 0 \quad \text{on } Y = 1 \quad (8)$$

$$\psi = 0, \quad \frac{\partial \theta}{\partial \xi} = 0 \quad \text{on } \xi = 0,$$

A physical quantity of interest in this problem is also the local Nusselt number  $Nu(\zeta)$  which is easily to be shown to be given by  $Nu(\xi) = -(\partial \theta / \partial Y)_{Y=0}$ .

## Results and discussion

Equations (6) and (7) along with the boundary conditions (8) were solved numerically for some values of the Rayleigh number  $R_a$  and viscosity parameter  $b$  using a finite-difference method similar to that used [8]. The aspect ratio parameter has been fixed at  $\varepsilon = 1$ . In order to test the accuracy of the present method, results for the minimum streamline  $\psi_{\min}$  in the case of constant viscosity ( $b=0$ ) are compared with those from [8] in table 1. It is seen that the agreement is very good so that we are confident that the present results are accurate. The effect

**Table 1**  
VALUES OF  $\psi_{\min}$  IN THE CASE OF CONSTANT VISCOSITY

$Ra$	$\psi_{\min}$	
	Merkin and Zang (1990)	Present results
0	-0.0402	-0.0397
1	-0.0406	-0.0401
50	-0.0596	-0.0594
100	-0.0562	-0.0560

of the viscosity parameter ( $b=0.6$  and  $2$ ) on the streamlines and isotherms are shown in figures 2 to 5 for  $R_a = 200$  and  $\varepsilon=1$ . We see from these figures that the streamlines and isotherms are sensible affected as  $b$  increases. Thus, the absolute value of  $|\psi_{\min}|$  is higher for  $b = 2$  than for  $b = 0.6$ . This fact is more evident for other values of the parameter  $b$ . Figure 6 shows the variation of the local Nusselt number  $Nu(\xi)$  with  $\xi$  for  $R_a = 200$  and some values of the parameter  $b$  with  $\varepsilon = 1$ . It is clear that the local Nusselt number is substantially affected by the variable viscosity of the fluid-saturated porous medium.

or inverse linear relations for viscosity with temperature as  $\mu = \mu^*(1 + b\theta)$ , or  $1/\mu = (1/\mu^*)(1 - b\theta)$ .

## Numerical method

It may be remarked that we encounter certain difficulties in solving equations (1) and (2) with the boundary conditions (3)-(5). Thus, to obtain an accurate numerical solution, the boundary conditions at infinity (5) need to be applied at a sufficiently large distance. In this respect, it is convenient to choose a transformation of the  $X$ - variable so that the solution domain may become of reasonable size. Such transformation has been suggested in [9] of the form  $\xi = \ln(1 + X)/\ln 2$  which maps  $X \in [0, 1]$  to  $\xi \in [0, 1]$  without too much change, but  $\xi > 1$  gives a relatively large value of  $X$  for relatively small values of  $\xi$ . In terms of the new variable  $\xi$ , equations (1) and (2) are subjected to the boundary conditions (3)-(5) which become:

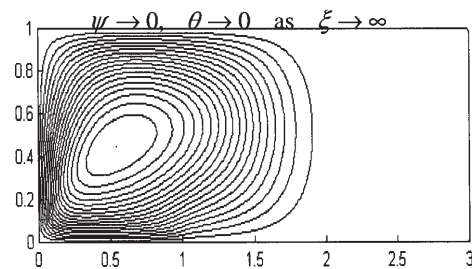


Fig. 2. Streamlines for  $R_a = 200$  and  $b = 0.6$  ( $\psi = -0.051012$ )

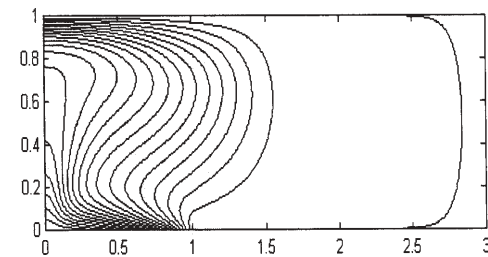


Fig. 3. Isotherms for  $R_a = 200$  and  $b = 0.6$

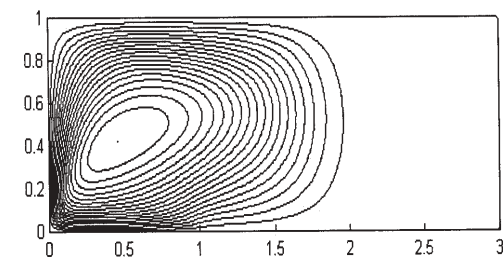


Fig. 4. Streamlines for  $R_a = 200$  and  $b = 2$  ( $\psi = -0.063409$ )

However, for the sake of space limitation, we have presented here results only for the heated case and several values of the parameter  $b$  with  $R_a = 200$  but there were obtained results for other values of the parameters  $R_a$  and

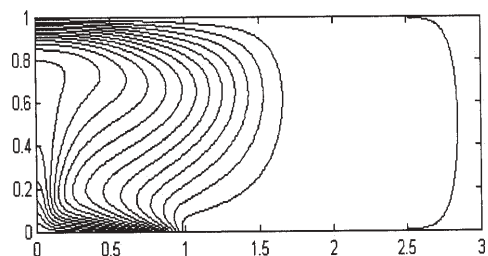


Fig. 5. Isotherms for  $Ra_a = 200$  and  $b = 2$

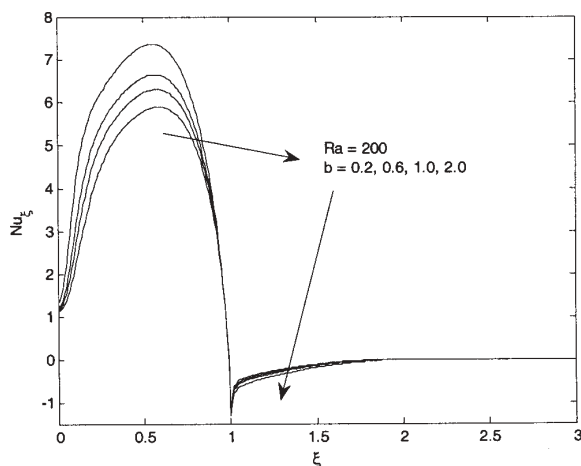


Fig. 6. Variation of the local Nusselt number for  $Ra_a = 200$  and  $b = 0.2, 0.6, 1, 2$

*b*. In addition, we obtained results also for the case when part of the bottom wall is cooled.

### Conclusions

A numerical study has been performed for the effect of variable viscosity on the steady free convection flow in a horizontal infinite channel filled with a fluid-saturated porous medium when a part of the bottom wall is heated.

Streamlines and isotherm as well as the local Nusselt number are shown graphically. Some values of the minimum streamlines are listed in a table. It is found that the flow and heat transfer characteristic are sensibly affected by the variable viscosity of the fluid-saturated porous medium.

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